An Integrated Approach to Large Amplitude Internal Wave Dynamics and Their Surface Signatures

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LONG-TERM GOALS

The long-term goal of this research is to develop an integrated model to describe the generation, propagation, and transformation of large amplitude internal solitary waves through interaction with bottom topography and to investigate sea surface roughness associated with the internal wave motions.

OBJECTIVES

The objective of this project is to investigate large amplitude internal wave dynamics and the associated surface roughness changes with developing an integrated internal wave prediction/remote sensing model based on (1) the strongly nonlinear internal wave model initialized with in-situ data and (2) the fully nonlinear surface wave model coupled with the surface current predictions of the internal wave model.

APPROACH

Our approach is to solve numerically the strongly nonlinear internal wave model for a two-layer system (Choi & Camassa, 1999) derived via an asymptotic expansion method under the sole assumption that the waves are long compared to the total water depth with including the leading order non-hydrostatic corrections. Once the surface current field induced by internal solitary waves is obtained from the numerical solution of the strongly nonlinear internal wave model, the evolution of surface gravity-capillary waves is studied using a surface wave prediction model that solves a closed system of nonlinear evolution equations (West *et al.*, 1987; Choi, 1995) for the free surface elevation and the velocity potential evaluated at the free surface.

WORK COMPLETED

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1. During the first year, we proposed a new regularized model for strongly nonlinear internal waves under the rigid-lid assumption that is free from the Kelvin-Helmholtz instability. This year, using local stability analysis under the assumption that the basic flow characteristics are locally constant, we identified a domain of physical parameters in which solitary waves are

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expected to be stable when the regularized model is solved. Then we validated numerically the result of our stability analysis; no sign of short wave instability was observed even when the internal wave amplitude reaches its maximum value. This result was reported in Choi *et al.* (2009, JFM, published).

2. To solve the two-dimensional regularized model effectively, an efficient iterative numerical scheme was developed and its convergence was examined theoretically. Its effectiveness was also tested numerically using a pseudo-spectral method and a number of numerical solutions of the model have been obtained, including the interaction between two solitary waves propagating obliquely. This result will be reported in a forthcoming publication in preparation (Choi *et al.*, 2009, in preparation).

3. We have studied more extensively the solitary wave solutions of an internal wave model with a free surface. Since the free surface internal wave model describes both the baroclinic (internal wave) and barotropic (surface wave) modes, finding the solitary wave solutions defined in a four-dimensional phase space is extremely complicated. Depending on physical parameters, there exist multi-humped solitary wave solutions that are absent in the rigid-lid model. A careful numerical scheme was developed to compute such solutions. In addition, the local stability characteristics were examined and the result was reported in Barros & Choi (2009, Studies in Applied Math, published).

4. For the deep-water configuration, the strongly nonlinear internal wave model has been extended to include the effects of higher-order nonlinearity and slowly-varying bottom topography of finite amplitude using a systematic asymptotic expansion. The model was discussed in de Zarate *et al.* (2009, Studies in Applied Math., published) and will be studied numerically in the near future.

5. For the shallow-water configuration, numerical solutions for a solitary wave propagating over slowly-varying bottom topography were obtained and compared favorably with laboratory experiments of Helfrich & Melville (1995). More detailed comparison with laboratory and field experiments will be reported in the future publication.

RESULTS

<u>Stability analysis of the regularized model and numerical verification</u>

Under the rigid lid assumption for the top boundary, the strongly nonlinear model has been regularized and can be written in terms of $\hat{u}_i(x,t)$, the horizontal velocity evaluated at the top and bottom boundaries for i=1 and 2, respectively, as

$$\eta_{i,t} + \left[\eta_i \ \left(\hat{u}_i - \tfrac{1}{6}\eta_i{}^2\,\hat{u}_{i,xx}\right)\right]_x = 0,$$
$$\hat{u}_{i,t} + \left(\frac{1}{2}\hat{u}_i^2 + g\zeta + \frac{P}{\rho_i}\right)_x = \left[\frac{1}{2}\,\eta_i{}^2\left(\hat{u}_{i,xt} + \hat{u}_i\hat{u}_{i,xx} - \hat{u}_{i,x}^2\right)\right]_x .$$

To better understand the stability characteristics of the model, we adopted a local stability analysis and identified the region of stability as shown in figure 1. Then, we demonstarted the robustness of the regularized model numerically, as shown in figure 2.

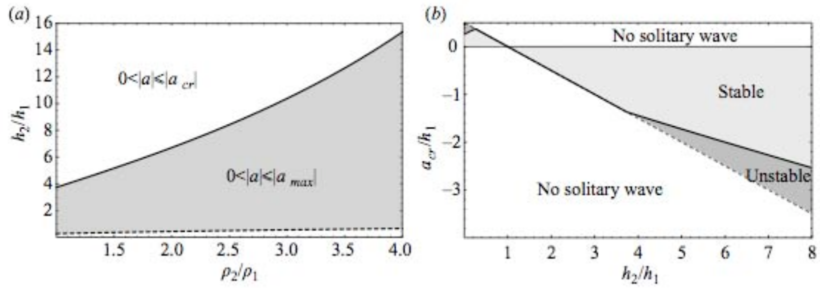


Figure 1. (*a*) Local stability charateristics of internal solitary waves: for the depth and density ratios inside the shaded region, the solitary waves are stable up to the maximum wave amplitude; otherwise, they are stable to the critical wave amplitude. (*b*) Critical wave amplitude versus depth ratio for a fixed density ratio.

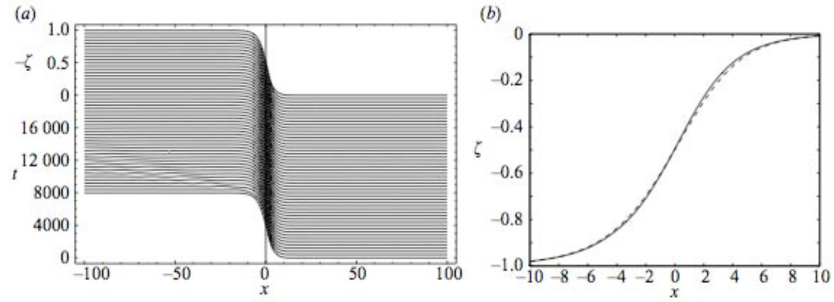


Figure 2. Numerical solution of the regularized model initialized with the front solution of the original stringly nonlinear model and no sign of the Kelvin-Helmholtz instability is observed even when the wave amplitude reaches its maximum value.

<u>**A new iterative scheme for the two-dimenional regularized model and numeirical solutions**</u>

To solve the strongly nonlinear model in two-horizontal dimensions, we developed a new iterative scheme with a set of tunable parameters and carried out a convrgence analysis to determine the parameters for fastest convergence. When we rewrite the regularized model as

$$\zeta_t = \left[\eta_1\left(u_1 - \frac{1}{6}\eta_1{}^2 u_{1,xx}\right)\right]_x,$$

$$V_t = \sum_{i=1}^{2}(-1)^i\rho_i\left[\frac{1}{2}u_i{}^2 + g\zeta - \frac{1}{2}\left(\eta_i{}^2 u_i u_{i,x}\right)_x - \frac{1}{6}\eta_i u_{i,x}\left(\eta_i{}^3 u_{i,xx}\right)_x\right]_x,$$

we solve these two evolution equations numerically. Then, we need to solve the following linear system to determine u_1 and u_2

$$\eta_1\left[u_1 - \frac{1}{6}\eta_1{}^2 u_{1,xx}\right] + \eta_2\left[u_2 - \frac{1}{6}\eta_2{}^2 u_{2,xx}\right] = C,$$

$$\rho_1\left[u_1 - \frac{1}{2}(\eta_1{}^2 u_{1,x})_x\right] - \rho_2\left[u_2 - \frac{1}{2}(\eta_2{}^2 u_{2,x})_x\right] = V.$$

but the inversion of the operators are computationally expensive. Therefore, we use the following iterative scheme:

$$\eta_1 U_1^{(n+1)} + \eta_2 U_2^{(n+1)} = R_1^{(n)},$$

$$\rho_1 U_1^{(n+1)} + \rho_2 U_2^{(n+1)} = R_2^{(n)},$$

from which u_i can be found easily by inverting a simple operator:

$$u_i^{(n+1)} = \left(1 - \alpha_i\, h_i^{\,2}\, \partial_x^2\right)^{-1} U_i^{(n+1)}$$

Notice that α_i need to be chosen for fastest convergence. Form Fourier analysis, α_i can be determined analytically and the result is shown in figure 3(*a*). Using this new iterative scheme, we solved the system numerically for a single elevation with doubly-periodic boundary conditions and the interaction between two obliquely propagating solitary waves, and the results are shown in figure 3(*b*) and 3(c), respectively.

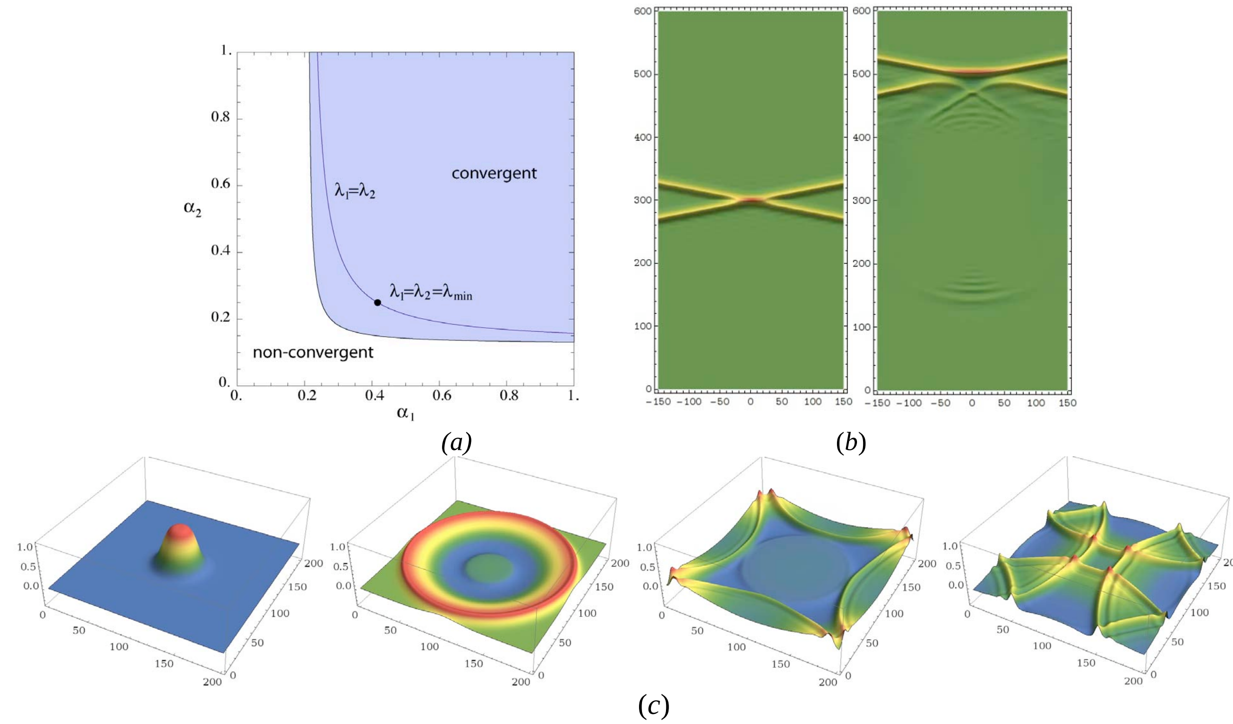
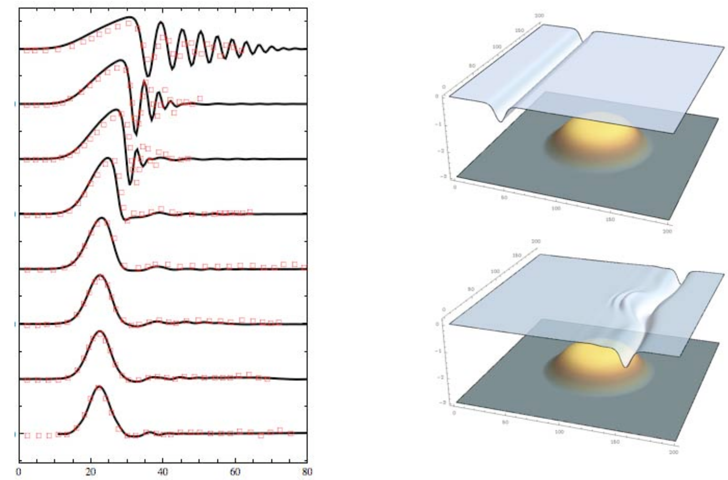


Figure 3. (*a*) Domain of convergence for the iterative scheme; (*b*) Interaction between two obliquely-propagating internal solitary waves. The amplitudes of two solitary waves are a/h_1=0.4 and the angle between them is π/16; (3) Evolution of a single elevation at the interface in time with periodic boundary conditions.

Interaction with bottom topography

The model in the presence of bottom topography is solved numerically and its solution is compared with the laboratory experiment of Helfrich & Melville (1985, JFM) for a single solitary wave propagating over a slope, as shown in Figure 4(a). Except for a slight difference observed when the solitary wave is at the top of the shelf, our numerical solution matches well with the experiment. Also a solitary wave propagating over bottom topography in the South China Sea has been considered numerically and the comparison of the numerical solution with field data is underway. Although our comparison with real data has focused on two-dimensional bottom topography, but three-dimensional bottom topography can be easily included in our numerical model, as shown in figure 4(b).



(*a*) (*b*)

Figure 4. (*a*) Comparison between our numerical solution (solid line) and experiments (symbols) by Helfrich & Melville (1985) for a single solitary wave propagating over a slope; (*b*) Propagation of a single solitary wave over a circular bump.

IMPACT/APPLICATIONS

The project is expected to provide a comprehensive tool for predicting and monitoring the strongly nonlinear internal wave activity and surface signatures.

RELATED PROJECTS

The PI of this project is working on the University of Michigan MURI project entitled “Optimum Vessel Performance in Evolving Nonlinear Wave Fields” and investigating the energy transfer and dissipation of short waves due to wave breaking whose understanding is important to radar remote sensing of internal solitary waves is being investigated.

PUBLICATIONS

Choi, W., Barros, R. and Jo, T.-C., A regularized model for strongly nonlinear internal solitary waves, J. Fluid Mech., 629, 2009, 73-85 [published, refereed].

Barros, R. and Choi, W., Inhibiting shear instability induced by large amplitude internal solitary waves in two-layer flows with free surface, Studies in Applied Mathematics, 122, 2009, 325-346 [published, refereed].

De Zarate, A. R., Vigo, D. G. A., Nachbin, A. and Choi, W. A higher-order internal wave model accounting for large bathymetric variations, Studies in Applied Mathematics, 122, 2009, 275-294 [published, refereed].

Barros, R. and Choi, W. On the hyperbolicity of two-layer flows. Proceedings of the FACM Conference, May 19-21, World Scientific, Singapore, ed. by D. Blackmore, A. Bose and P. Petropoulos, 95-103, 2008. [published, non-refereed]